Open Problem Session at GROW 2019

Vienna, 23rd September 2019

Problem 1 - Jan Kratochvíl
What is the complexity (P or NP) of the following problem?

Given a graph $G = (V, E)$, decide if there exist a partition $V = A \dot{\cup} (V \setminus A)$ where the following is forbidden: There is some $X \subseteq V$ with $|X| = 4$ such that $G[A \cap X]$ and $G[(V \setminus A) \cap X]$ are (possibly empty) cliques and $G[X]$ has exactly these two cliques as connected components.

Problem 2 - Thekla Hamm
What is the complexity (P or NP) of the following problem?

Given a chordal undirected graph, count the number of so called *moral acyclic orientations* (MAOs for short). An acyclic orientation is moral if the edges of an induced P_3 (a path with three vertices) are not both directed towards the middle vertex of this P_3.
Known results:

- $\mathcal{O}(2^n)$-algorithm for n-vertex graphs [TK19].
- Polytime algorithm for graphs of bounded treewidth [TK19].
- Polytime algorithm for graphs of bounded vertex degree [GSKZ19].
- Polytime algorithm for graphs that have a clique tree that has a polynomial (in the number of vertices of the graph) number of subtrees.

[GSKZ19] AmirEmad Ghassami, Saber Salehkaleybar, Negar Kiyavash, and Kun Zhang. Counting and sampling from markov equivalent dags using clique trees. In *The Thirty-Third AAAI Conference on Artificial Intelligence, AAAI 2019, Honolulu, Hawaii, USA, January 27 - February 1, 2019.*, pages 3664–3671, 2019.

[TK19] Topi Talvitie and Mikko Koivisto. Counting and sampling markov equivalent directed acyclic graphs. In *The Thirty-Third AAAI Conference on Artificial Intelligence, AAAI 2019, Honolulu, Hawaii, USA, January 27 - February 1, 2019.*, pages 7984–7991, 2019.

Problem 3 - Mamadou Kanté
Consider the problem #DOM of counting the number of minimal dominating sets of a graph. Even in split graphs this problem is known to be #P-complete (essentially listing all minimal dominating sets is the best one can do).

A *k-sun* is the graph that arises from a clique on vertices $v_1, \ldots, v_k$ by adding k vertices $w_1, \ldots w_k$ and edges between w_i and v_i and w_i and v_{i+1} (v_1 in the case of $i = k$).

A graph is *recursively chordal* if it is k-sun-free for $k \geq 4$. In particular *stongly chordal* graphs, which are k-sun-free for $k \geq 3$, are recursively chordal.

For a hereditary graph class $\mathcal{C}$ of chordal graphs, prove (or disprove): #DOM for $\mathcal{C}$ is in P if and only if $\mathcal{C} \subseteq$ class of recursively chrodal graphs. (A first step is already known: #DOM for strongly chordal graphs is in P.)

Problem Set 4 - Steven Chaplick

A *bundled crossing* in a drawing of a graph is a group of crossings between two sets of locally pairwise non-crossing edges. (For a formal definition see e.g. [CvDK+18].) The *bundled crossing number* of a drawing is the minimum number of bundled crossings that group all crossings in a drawing of the graph.

The *bundled crossing number* bc of a graph is the minimum bundle crossing number of a simple drawing of this graph. The *circular bundled crossing number* of a graph bc° is the minimum bundle crossing number of a simple drawing this graph in which the vertices of the graph are required to lie on the boundary of a disk and all edges of the graph are required to lie inside this disk. Deciding $\mathrm{bc}^\circ \leq k$ is in FPT[CvDK+18].

4.1 Is there a faster FPT-algorithm for deciding $\mathrm{bc}^\circ \leq k$ (in particular one that avoids the use of Courcelle's theorem)?

4.2 Is deciding $\mathrm{bc}^\circ \leq k$ even NP-hard?

4.3 Is deciding $\mathrm{bc} \leq k$ in FPT for general simple layouts?

[CvDK+18] Steven Chaplick, Thomas C. van Dijk, Myroslav Kryven, Ji-won Park, Alexander Ravsky, and Alexander Wolff. Bundled crossings revisited. *CoRR*, abs/1812.04263, 2018.

Problem Set 5 - George Mertzios

A *temporal graph* with *lifetime* $T \in \mathbb{N}$ is a tuple (G, λ) of a graph $G = (V, E)$ and a function $\lambda : E \to 2^{\{1,\ldots,T\}}$. For some $\Delta \leq T$, *sliding Δ-time windows* are defined as the intervals $W_t = [t, t + \Delta - 1]$ for $t \in [1, T - \Delta + 1]$.

A solution to Δ-TVC (sliding Δ-time window temporal vertex cover)[AMSZ19] is a minimum set $X \subseteq V \times [1, T]$ such that for every sliding Δ-time window W_t and every edge $e \in \bigcup_{t' \in W_t} \lambda^{-1}(t')$ there is some $(v, t') \in X$ such that $t' \in W_t$, $t' \in \lambda(e)$ and v is an endpoint of e.

5.1.1 What is the complexity of solving Δ-TVC on degree-at-most-2 temporal graphs?

5.1.2 Can Δ-TVC on general graphs be approximated within a factor better than 2Δ?

5.1.3 Can Δ-TVC on always-degree at most d temporal graphs be approximated within a factor better than d?

A solution to Temporal Matching [MMN+19] is a maximum set $M \subseteq E \times [1, T]$ such that for every sliding Δ-time window W_t and every vertex $v \in V$ there is at most one $(e, t) \in M$ such that $t \in W_t$, $t \in \lambda(e)$ and v is an endpoint of e.
Temporal Matching is APX-hard, even for $T = 3$ and $\Delta = 2$.

For $T = 3$ and $\Delta = 2$, an easy (2/3)-approximation is given by taking the maximal of the maximum matchings in $(V, \lambda^{-1}(i) \cup \lambda^{-1}(j))$ for $i, j \in \{1, 2, 3\}$. This generalises to a $\frac{\Delta}{2\Delta-1}$-approximation for general T and Δ.

5.2 Is there a polytime approximation algorithm for Temporal Matching with approximation ratio at least $\frac{1}{2} + \varepsilon$?

[AMSZ19] Eleni C. Akrida, George B. Mertzios, Paul G. Spirakis, and Viktor Zamaraev. Temporal vertex cover with a sliding time window. *Journal of Computer and System Sciences*, 2019.

[MMN+19] George B. Mertzios, Hendrik Molter, Rolf Niedermeier, Viktor Zamaraev, and Philipp Zschoche. Computing maximum matchings in temporal graphs. *CoRR*, abs/1905.05304, 2019.

Problem Set 6 - Jakub Gajarský
Given a graph G and a FO-formula $\psi(x, y)$, define $I_\psi(G)$ as the graph $(V(G), \{\{u, v\} \mid G \models \psi(u, v)\})$. Let $\mathcal{C}$ be some sparse graph class.

6.1 Given some FO-formula $\psi(x, y)$, characterise $I_\psi(\mathcal{C}) = \bigcup_{G \in \mathcal{C}} I_\psi(G)$.

6.2 Given $H \in I_\psi(\mathcal{C})$, compute $G \in \mathcal{C}$ such that $H = I_\psi(G)$.

6.3 (Weaker variation of 6.2) Show that for every FO-formula $\psi(x, y)$ there exists a class $\mathcal{C}'$ of sparse graphs, a FO-formula $\psi'(x, y)$ such that $|\psi'| \le |f(|\psi|)|$ and a polytime algorithm A such that, given $H \in I_\psi(\mathcal{C})$, A computes $G \in \mathcal{C}'$ such that $H = I_{\psi'}(G)$. Ideally, A should run in FPT-time parameterised by $|\psi|$, but XP is also fine.

Some known results:

- For $\mathcal{C}$ as the class of bounded degree graphs, both 6.1 and 6.2 are resolved [2].
- 6.3 is answered for map graphs by transforming them into nowhere dense graphs [1].
- For graphs of bounded expansion 6.1 is resolved [3].

[1] Kord Eickmeyer and Ken-ichi Kawarabayashi. Fo model checking on map graphs. In Ralf Klasing and Marc Zeitoun, editors, *Fundamentals of Computation Theory*, pages 204–216, Berlin, Heidelberg, 2017. Springer Berlin Heidelberg.

[2] J. Gajarský, P. Hliněný, J. Obdržálek, D. Lokshtanov, and M. S. Ramanujan. A new perspective on fo model checking of dense graph classes. In *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science*, LICS '16, pages 176–184, New York, NY, USA, 2016. ACM.

[3] Jakub Gajarský, Stephan Kreutzer, Jaroslav Nesetril, Patrice Ossona de Mendez, Michal Pilipczuk, Sebastian Siebertz, and Szymon Torunczyk. First-order interpretations of bounded expansion classes. In *45th International Colloquium on Automata, Languages, and Programming, ICALP 2018, July 9-13, 2018, Prague, Czech Republic*, pages 126:1–126:14, 2018.

Problem Set 7 - Robert Ganian

An *ℓ-page book embedding* of a graph consists of a linear ordering of its vertices which induces an embedding of the vertices along a line (also called *spine*) and an embedding of its edges into at most ℓ halfplanes that have the spine as their boundary, such that the embeddings into each halfplane are planar.

Is finding an ℓ- page book embedding of a graph FPT parameterised by...

7.1 ...the treewidth of G?

7.2 ...the treedepth of G?

7.3 ...the minimum size of a feedback edge set of G?

All these questions are interesting even if the ordering of the vertices is fixed.
Known: Finding an ℓ-page book embedding is in FPT parameterised by the minimum size of a vertex cover of G [1].

[1] Sujoy Bhore, Robert Ganian, Fabrizio Montecchiani, and Martin Nöllenburg. Parameterized algorithms for book embedding problems. *CoRR*, abs/1908.08911, 2019.

Problem Set 8 - Christophe Paul (originally posed by Maurice Pouzet)

Given a tournament T an *inversion* in T is the operation of selecting a vertex set $S \subseteq V(T)$ and reversing every arc in $T[S]$. The minimum number of inversions needed to transform T into a transitive tournament is called *inversion index* of T and denoted by $\text{Inv}(T)$.

8.1 Is deciding $\text{Inv}(T) \leq k$ NP-complete?

8.2 Is deciding $\text{Inv}(T) \leq k$ in FPT? (Membership in XP is known by a WQO argument.)

8.3 Is it true that if T can be decomposed into T_1 and T_2 such that all arcs between T_1 and T_2 are directed from T_1 to T_2, then $\text{Inv}(T) = \text{Inv}(T_1) + \text{Inv}(T_2)$?

8.4 What can one say about the problem, if one restricts the cardinality of vertex sets one can choose for inversion?