6th Workshop on Graph Classes, Optimization,
and Width Parameters

Santorini Island, October 09–11, 2013

List of Open Problems

Yixin Cao: What is the complexity of finding a *hole* (induced cycle with at least 4 vertices) in an undirected graph G? What is the complexity of finding an *odd hole* in an undirected graph G?

Known results: Finding triangles in an undirected graph G can be done in time $O(n^4)$ and there is a polynomial time algorithm for finding even holes.

Isolde Adler: We define the problem:

Undirected k Disjoint Paths	
Input:	An undirected graph G, a collection $T = \{(t_1, s_1), \ldots, (t_k, s_k)\}$ of pairs of vertices in G.
Parameter:	k
Question:	Are there k vertex-disjoint-paths connecting the pairs (t_i, s_i) in G?

Find an algorithm solving Undirected k-Disjoint Paths problem in time $2^{O(k)}\mathsf{poly}(|V(G)|)$.

Known results: There is a $2^{2^{O(k)}} \cdot \mathsf{poly}(|V(G)|)$ time algorithm solving Undirected k-Disjoint Paths *problem.*

Nicolas Trotignon: We will use the following definition:

Definition. *A hereditary class of graphs is* χ-bounded *if there exists a function* f *such that for every graph* G *in the class:* $\chi(G) \leq f(\omega(G))$.

Suppose that there exists a polynomial time algorithm to compute the stable set number for all graphs in a class $\mathcal{C}$. Then is $\mathcal{C}$ χ-bounded?
Note: One such class is the class of claw-free graphs.

Martin Golumbic: Find a characterization for the permutation graphs that are B_0.

Known results: The B_1 *graphs are precisely the permutation graphs. Also, a cograph is* B_0 *if and only if it is* W_4*-free.*

Frédéric Mazoit: Given an ordered set $(X, \leq)$ we define another ordered set $(2^X, \leq)$ by:

$$A \leq B \text{ iff } \forall a \in A, \exists b \in B \text{ such that: } a \leq b.$$

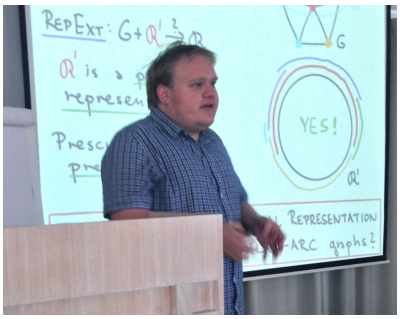
If X is the class of planar graphs, is it true that $(2^X, \text{minor})$ is well-quasi-ordered?

Pavel Klavík: What is the complexity of the following problem:

Partial Representation Extension-Circular-Arc Graphs	
Input:	A partial representation of a circular-arc graph G.
Question:	Can the given representation be extended to a full representation of G?

For more details, see:
http://pavel.klavik.cz/research/pres/grow2013_open_problem_extending_circular_arc.pdf

Martin Milanič: We will use the following definition:



Definition. *A graph G is* CIS *if for every maximal cique C and for every maximal stable set S, $C \cap S \neq \emptyset$.*

What is the complexity of recognizing if a graph G is CIS?

Marcin Pilipczuk: We will use the following definition:

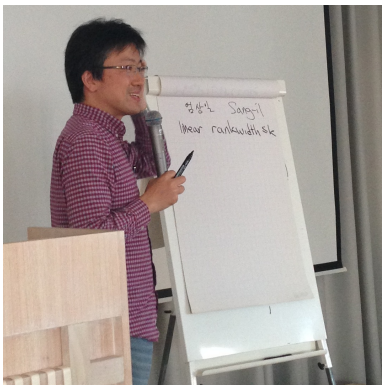
Definition. *A graph G is called* mixed *if it contains both directed and undirected edges.*

Is there an FPT(k) for the following problem?

STEINER ORIENTATION	
Input:	A mixed graph G, a collection $T = \{(t_1, s_1), \ldots, (t_k, s_k)\}$ of pairs of vertices in G.
Parameter:	k
Question:	Can we orient the edges of G in such a way that there exist k paths connecting the pairs (t_i, s_i) in G after the orientation?

Sang-Il Oum: Find a "good" FPT-algorithm for deciding if a graph G has linear rankwidth $\leq k$.

Note: We know that there exists an FPT*-algorithm for this problem from Courcelle's theorem that is why we emphasize on "good".*



Flavia Bonomo: We will use the following definition:

Definition. *A graph $G = (V, E)$ is k-thin if there exist an ordering $<$ of the vertices of G and a partition of V into k sets with the property that, for each triple of vertices r, s, t with $r < s < t$, if r and s are in the same partitioned set and $\{t, r\}$ is an edge, then $\{t, s\}$ is an edge.*

Is there a characterization for the 2-thin graphs?

Known results: The 1-thin graphs are precisely the interval graphs.

Victor Chepoi: Let $\rho_R(G)$ be the minimum number of balls of radius R covering a finite graph $G = (V, E)$ and let $\gamma_R(G)$ be the maximum number of pairwise disjoint balls of radius R of G.

Question 1. Is it true that $\rho(G) \leq c\gamma(G)$ for some universal constant, all radii R, and all planar graphs G? [A positive answer to Question 1 would be obtained by solving the following question.]

Question 2. (the weak doubling property) Is it true that any positive R, any planar graph G contains a ball of radius $2R$ which can be covered with a constant number of balls of radius R ?

Mamadou Kanté: Can the graph isomorphism problem be solved in polynomial time for graphs of bounded linear rank-width?

Andreas Brandstädt : What is the complexity of finding a maximum independent set for hole-free graphs, i.e., graphs without induced cycles of length at least 5?

Michał Pilipczuk: Suppose that Π is an induced-hereditary graph class and recognizable in polynomial time. Is it true that for every such class Π there is a constant $c_\Pi < 2$ and an algorithm for frinding an induced Π-subgraph in a graph G in $\mathcal{O}^*(c_\Pi^n)$-time?